Homework 1 – 1.1

Intermediate Value Theorem (IVT):

If and is any number between and , then there exists a number in for which .

Rolle’s Theorem:

Suppose and is differentiable on . If , then a number in exists with .

# Problem 1

Show that the following equations have at least one solution in the given interval.

Using the following C++ program:

//Headers

#include <iostream>

#include <cmath>

using namespace std;

//This is the function that we will use to solve the problem

void function(double x)

{

double fx = 0;

fx = x \* cos(x) - 2 \* pow(x, 2) + 3 \* x - 1;

cout << x << " " << fx << endl;

}

int main()

{

//Here we will take the value x that we will use to find f(x)

double value1 = .2, value2 = .3, value3 = 1.2, value4 = 1.3;

function(value1);

function(value2);

function(value3);

function(value4);

return 0;

}

We see that at the following values are given for each end point.

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Using the IVT, since and , we know that there has to be at least one value of such that .

# Problem 4

Find max for the following function and intervals.

Using a program:

#include <iostream>

#include <cmath>

#include <math.h>

using namespace std;

//This is the function that is in question, I will have it return the value so that it can be tested.

double function(double x)

{

double fx = 0;

fx = ((2 - exp(x) + 2 \* x) / 3);

return fx;

}

int main()

{

//X is going to be the value that will be tested in the loop and delx will be the small increment of x.

//Xmax and xmin are going to be where the Largest and smallest f(x) is at. They will both start at 0.

double X = 0, delx = 0.00001, Xmax = 0, xmin = 0;

//THis do loop will go through the range from 0 to 1 via increments of delx

do

{

//Here the value will be tested to see which x gives the Largest f(x) and updated if needed.

if (function(X) > function(Xmax))

{

Xmax = X;

}

//Here the value will be tested to see which x gives the smallest f(x) and updated if needed.

if (function(X) < function(xmin))

{

xmin = X;

}

//This is where the X value will be increased so that all the range can be tested.

X = X + delx;

} while (X <= 1);

//This is where the Largest and smallest values will be outputted.

cout << "The Maximun value is found at: x = " << Xmax << ", f(x) = " << function(Xmax) << endl;

cout << "The minimun value is found at: x = " << xmin << ", f(x) = " << function(xmin) << endl;

return 0;

}

From this program we get that the Maximum value occurs at giving . The minimum value is found at giving

Using Calculus to verify our results:

Testing this point and the endpoints:

Thus our numeric solutions seem to coincide with the analytic solutions.

# Problem 5

Use the Intermediate Value Theorem 1.11 and Rolle’s Theorem 1.7 to show that the graph of crosses the x-axis exactly once, regardless of the value of the constant k.

Since is a polynomial function, we know that it is continuous in the entire real domain.

Note that:

Via the IVT, we see that there must be at least one value in the domain such that , .

To show that there is only one value of where the function crosses the x-axis we must find the Maximum and minimum values. To do this, we need to know where the derivative is equal to .

Since we see that there are no real solutions to this condition regardless of value , Rolle’s Theorem gives us that there are no two values such that (This only works if ). Therefore, there exists only one value where .